probability

Adam Okulicz-Kozaryn

adam.okulicz.kozaryn@gmail.com

this version: Wednesday 20th November, 2024 07:28

outline

intuition and why bother?

computing probability

conditional probability and independence

outline

evolutionary and counterintuitive!

- evolution made us survive in environment that is long time gone!
  - our cognitive function and probability calculation off!
- we need stats and probability to help us think!
- eg: overestimate prob of memorable/flashy events
  - terrorist attack and airplane crash are similar to mistaking stick for snake etc–better be extra careful and see even if it's not there
- underestimate much more deadly sugar and fat (which were always rare and desirable)

making right decisions

- people and orgs make mistakes bc miscalculate prob
  - gambling and lotteries
  - smoking (hundreds or thousands of % increased risks!)
  - flying v driving, etc
  - 9/11 killed extra thousands bc ppl chose to drive (Wheelan, 2013, p.72-3)
- think about probabilities when making a decision
- the easiest (but already informative and helpful):
  - $\frac{occurences}{total}$; eg : $\frac{cancers}{smokers}$; $\frac{crashes}{miles}$; $\frac{crashes}{hours\ travelled}$
  - depends how you measure eg miles v hours !

why is prob relevant to MPA student?

- probability may be confusing
- but it does improve thinking/decision making
  - it's everywhere
- usually don't realize it, but we calculate prob all the time
  - if can do it better, it'd help our lives enormously!

important for organizations

- eg identify teachers who cheat or doctors who overcharge
  - just identify outliers, and unlikely events (nontypical)
- often non-intuitive

  eg swimming pools kill more kids than guns
  - (just count hh with pools and guns and kids deaths)
- many interesting examples in Levit's `freakonomics.com`

i used to allow undocumented emergencies

- not anymore! thank god for probability!!!
- out of 7 students 4 had their grandmas die
- the reported probability too high to be plausible!
- what's the prob of grandma dying this semester?
  - say avg grandma expected to last at least (if not more) 10 years, or 40 three-month periods in a year (about semester long)
  - so about $1/40$, so for 10 students class: every 4 semesters one grandma dead

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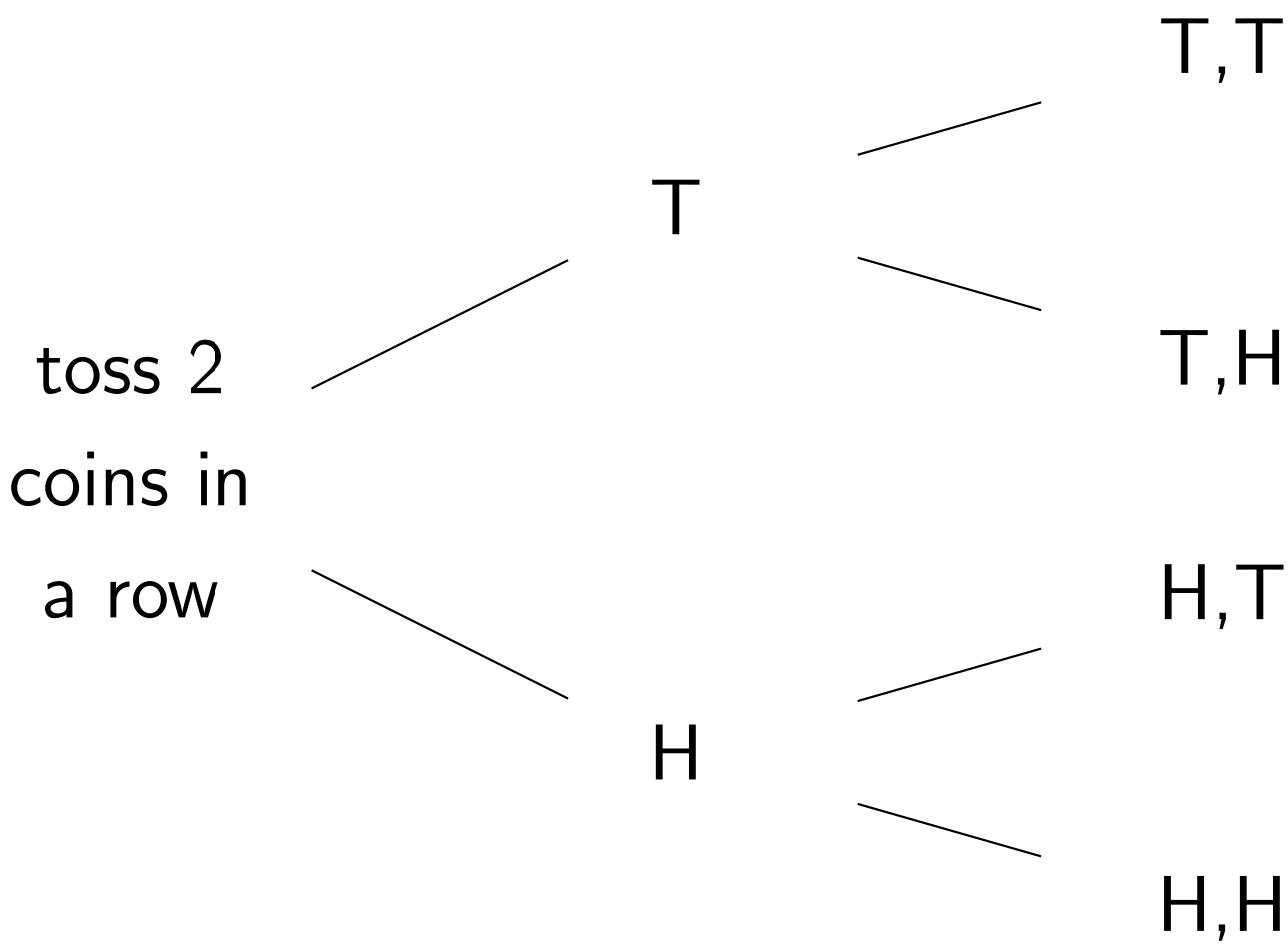
what is it?

- likelihood that the event will occur
  (the proportion of times the outcome would occur)
- ranges from 0 to 1
- 0 means impossible
  - almost never 0, almost nothing is impossible
- 1 means certain
  - also almost never 1; almost nothing is certain

tree



table

HH	HT
TH	TT

- 1st row H in first flip
- 2nd row T in first flip
- 1st column H in second flip
- 2nd column T in second flip

exercises

- what is the probability of getting 2 T in 2 flips ?
- we just showed with tree and table that there are 4 possible events, and only one outcome with 2 T, so $P = \frac{1}{4}$
- how about at least 1 T ?
- $P = \frac{3}{4}$
- how about exactly 1 T ?
- $P = \frac{2}{4}$

cards examples

- $P(heart) = \frac{1}{4}$
- mutually exclusive (disjoint)

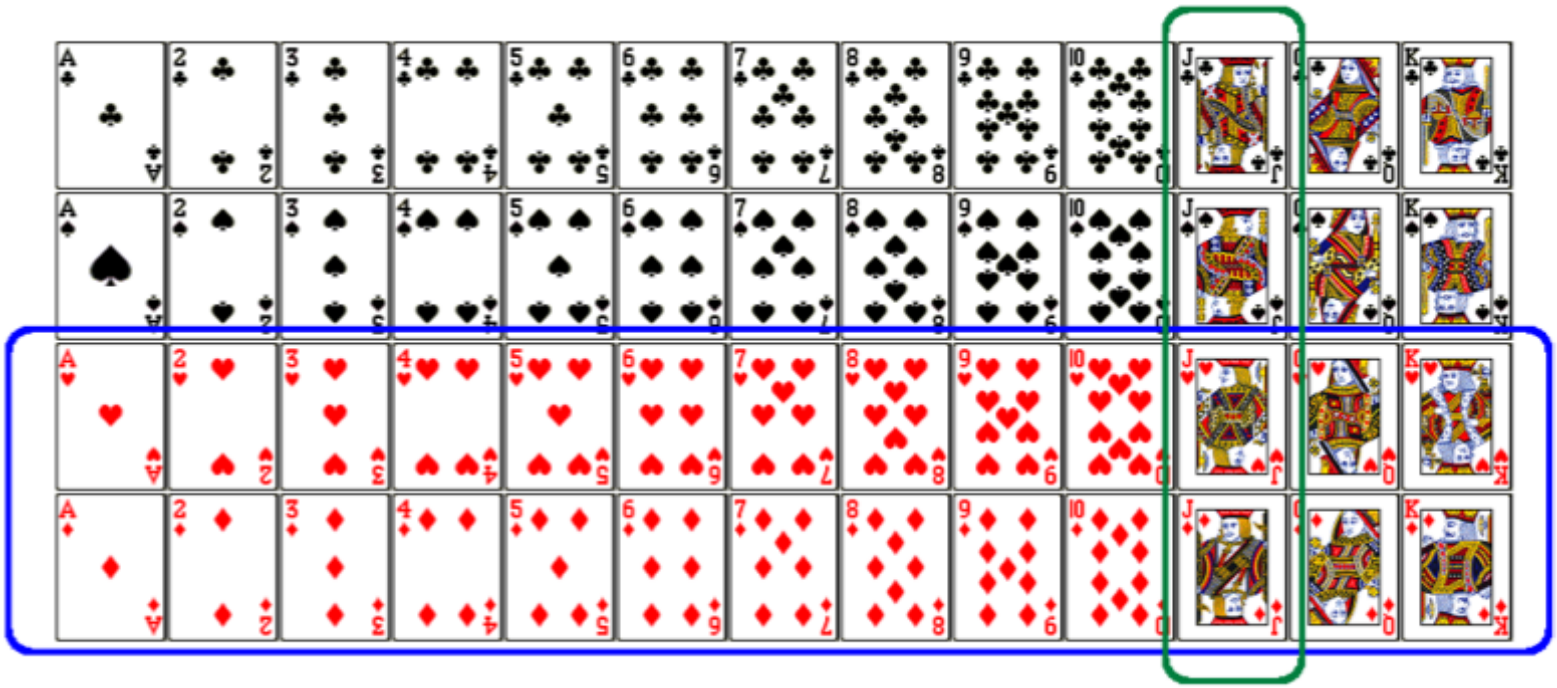
  $P(A \cup B) = P(A) + P(B)$
  - $P(ace\ or\ king) = P(ace) + P(king) = \frac{1}{13} + \frac{1}{13} = 2/13$
- not mutually exclusive (non-disjoint)

  $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
  - $P(ace\ or\ black) = P(ace) + P(black) -$

    $P(ace\ and\ black) = \frac{4}{52} + \frac{26}{52} - \frac{2}{52} = 7/13$
- $P(heart|red) = 1/2$ because sample space is reduced to 26 red cards (will get back to it at the end!)

Union of non-disjoint events

What is the probability of drawing a jack or a red card from a well shuffled full deck?



$$P(\textit{jack or red}) = P(\textit{jack}) + P(\textit{red}) - P(\textit{jack and red})$$

$$= \frac{4}{52} + \frac{26}{52} - \frac{2}{52} = \frac{28}{52}$$

Figure from http://www.milefoot.com/math/discrete/counting/cardfreq.htm

What is the probability that a randomly sampled student thinks marijuana should be legalized or they agree with their parents' political views?

	Share Parents' Politics		
Legalize MJ	No	Yes	Total
No	11	40	51
Yes	36	78	114
Total	47	118	165

(a) (40 + 36 - 78) / 165

(b) (114 + 118 - 78) / 165

(c) 78 / 165

(d) 78 / 188

(e) 11 / 47

Product rule for independent events

P(A and B) = P(A) x P(B)

Or more generally, P(A1 and … and Ak) = P(A1) x … x P(Ak)

You toss a coin twice, what is the probability of getting two tails in a row?

P(T on the first toss) x P(T on the second toss)

= (1 / 2) x (1 / 2) = 1 / 4

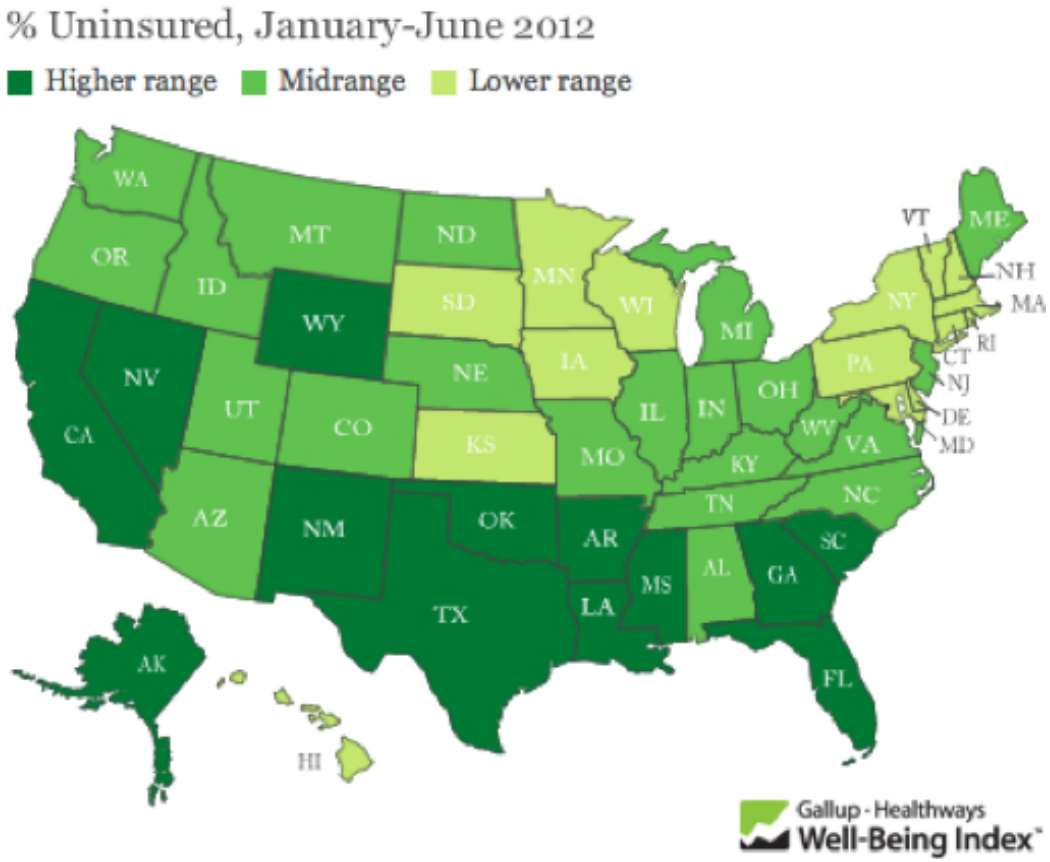
A recent Gallup poll suggests that 25.5% of Texans do not have health insurance as of June 2012. Assuming that the uninsured rate stayed constant, what is the probability that two randomly selected Texans are both uninsured?

(a) 25.5^2

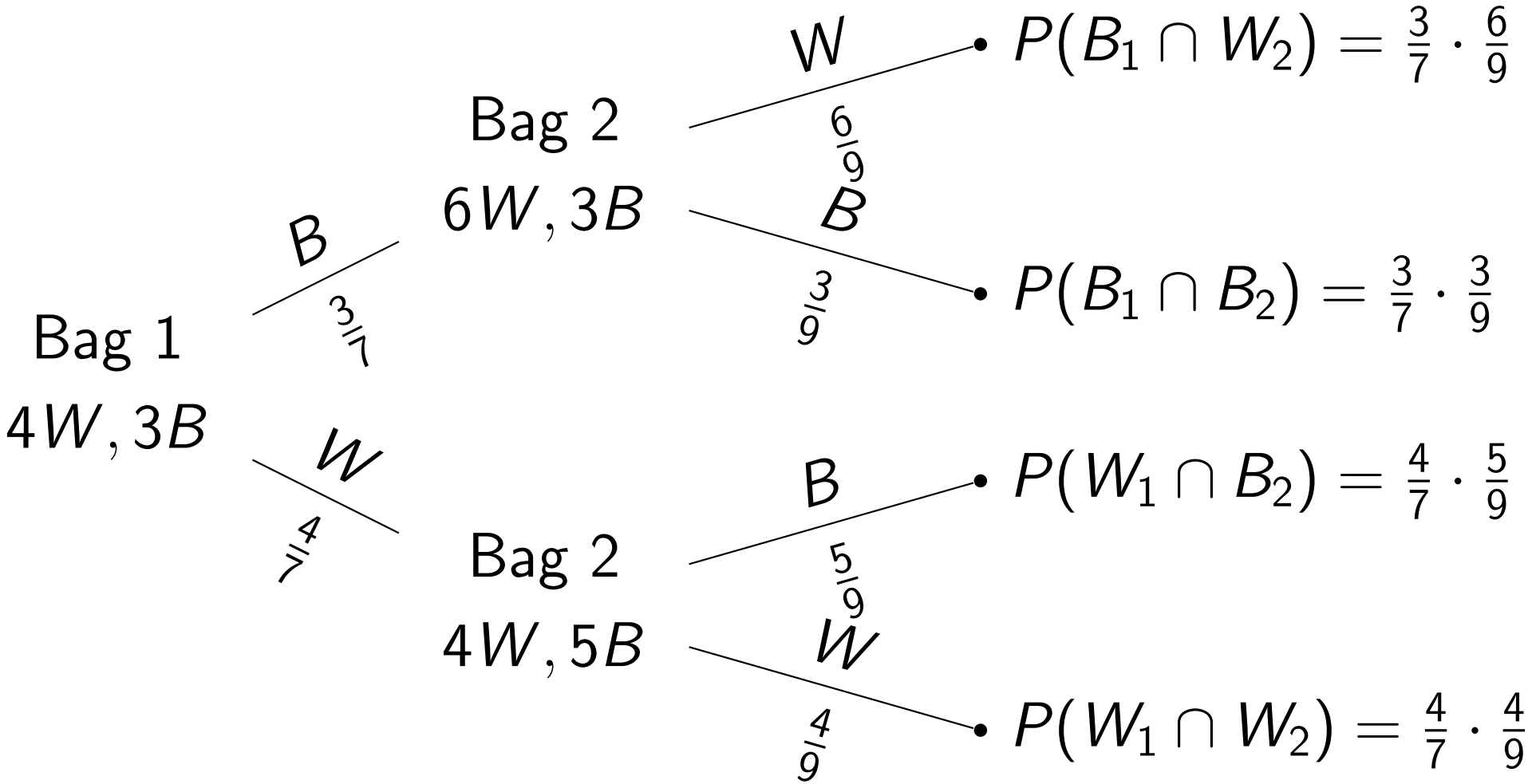
(b) 0.255^2

(c) 0.255×2

(d) $(1 - 0.255)^2$



tree example



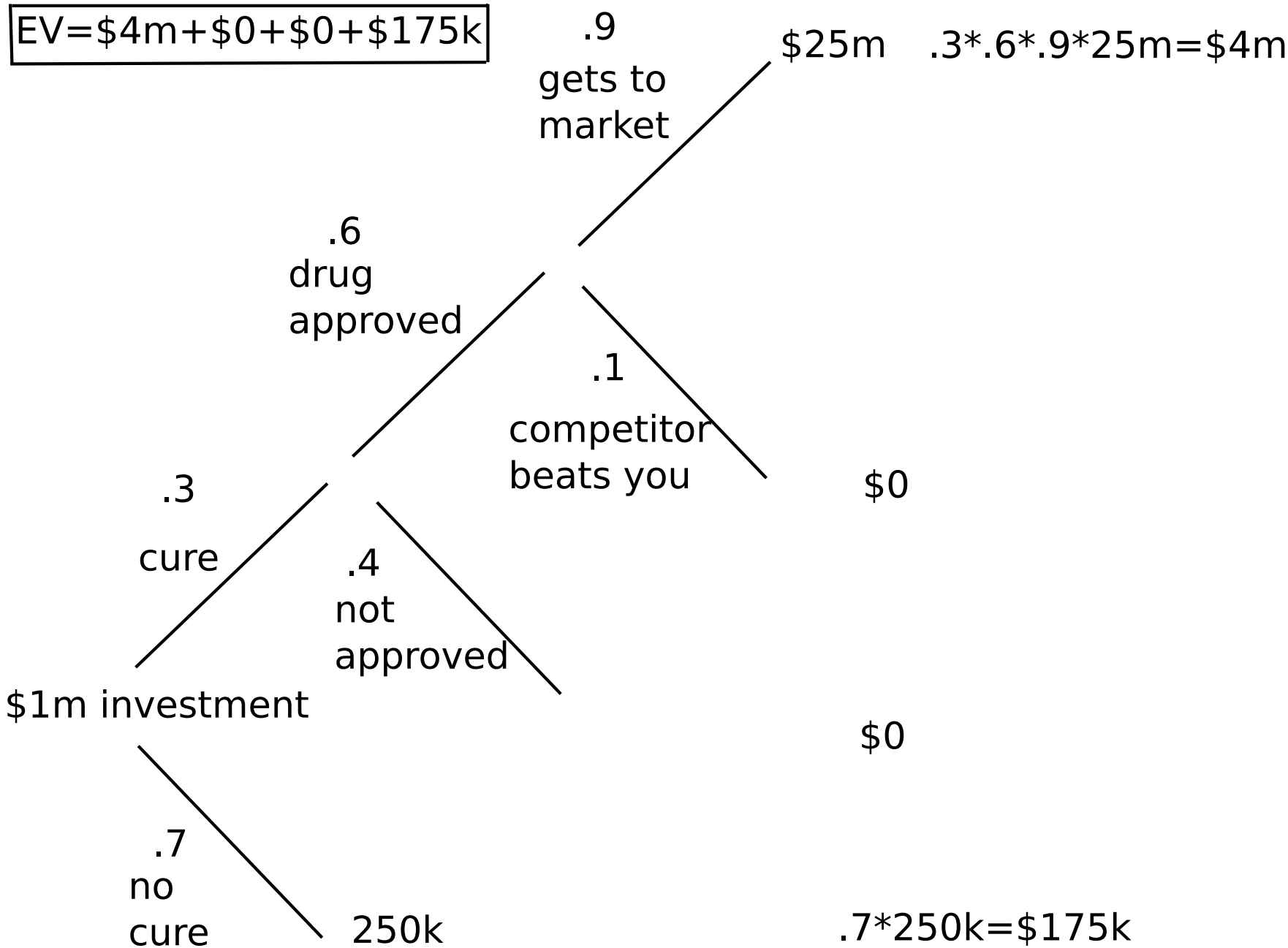
http://www.onemathematicalcat.org/Math/Algebra_II_obj/prob_tree_diagrams.htm

probability tree useful in pub adm

- say a local nonprofit such as LAEDA organizes entrepreneurship workshops
- high prob a person from neighb attends .7
- moderate prob person finishes the course .4
- low prob graduate actually applies the skills in the real world .1
- very low prob graduate succeeds .01

Expected Value (Wheelan, 2013, p83)

- just multiply value ($ amount) by associated probability AND add them up
- and this is how much you are expected to get on average


EV=$4m+$0+$0+$175k
.9
gets to market
$25m
.3*.6*.9*25m=$4m
.6
drug approved
.1
competitor beats you
$0
.3
cure
.4
not approved
$1m investment
$0
.7
no cure
250k
.7*250k=$175k

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conditional probability

- $P(A|B) = \frac{P(A \cap B)}{P(B)}$
- you have $P(A \cap B)$ in numerator because both A and ($\cap$) B need to happen to be conditional on B, if A happens but not B, then it cannot be conditional on B

table practice (all numbers in the body are "$\cap$")

	Type of Policy (%)			
Category	**Fire**	**Auto**	**Other**	**Total %**
Fraudulent	6	1	3	10
Nonfraudulent	14	29	47	90
Total	20	30	50	100

- $P(\mathit{fire}) = \frac{20}{100} = .2$
- $P(F|\mathit{fire})$ is $6/20$ or $.06/.2$
- $P(\mathit{fire}|F)$ is $.06/.1$

strategy

- probability can be confusing
- but if you think about it, you'll figure it out
- formulas may be more confusing than revealing
- use formulas, eg $\frac{occurences}{total}$($\frac{cancers}{smokers}$ etc), tables or trees

Levitt, S. D. and S. J. Dubner (2010): Freakonomics, vol. 61, Sperling & Kupfer.

Wheelan, C. (2013): Naked statistics: stripping the dread from the data, WW Norton & Company.